Transient Growth of Damped Baroclinic Waves

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(Manuscript received 26 February 1985, in final form 24 June 1985)

ABSTRACT

A solution of the linear initial value problem for the model of Eady with the inclusion of Ekman damping is presented. This model exhibits large transient growth of perturbations for synoptic cyclone spatial scales and a realistic value of the vertical turbulent viscosity coefficient despite the fact that all normal modes are exponentially decaying. Similar results are found for the model of Charney, implying that exponential instability cannot, in general, serve to explain the occurrence of cyclone scale disturbances in midlatitudes. Rather these are seen to arise additionally and perhaps predominantly from the release of mean flow potential energy by favorably configured initial perturbations. The Petterssen criterion for midlatitude cyclogenesis results naturally from this development as does its extension to the formation of subtropical monsoon depressions. Implications for the maintenance of midlatitude temperature gradients are discussed.

1. Introduction

Our present understanding of the dynamics of synoptic scale cyclones is based on quasi-geostrophic theory, particularly the work of Charney (1947) and Eady (1949) who found that the equation expressing conservation of quasi-geostrophic pseudo-potential vorticity in a vertically sheared mean flow, together with appropriate boundary conditions, results in eigenmode solutions resembling midlatitude cyclones in scale and structure. Furthermore, for some spatial scales and mean flow parameters there exist associated eigenvalues corresponding to an exponential increase in perturbation amplitude with time. The explanation for cyclogenesis which has emerged is that a broad spectrum of infinitesimal modal excitations results in the eventual dominance of the mode with the most rapid exponential growth, and its scale and structure determine the disturbance field at finite amplitudes.

While some features of this theory are supported by synoptic experience, e.g., the horizontal scales of a few thousand kilometers and a westward tilt with height of the growing pressure perturbation, other features correspond less well. In particular, synoptic experience as expressed in forecast rules emphasizes particular fairly large amplitude initial perturbations that evolve in structure as the deepening proceeds (Petterssen, 1955; Palmen and Newton, 1969). These observations can be reconciled with quasi-geostrophic theory by introducing the continuous spectrum of singular neutral modes which complement the eigenmodes and allow solution of the initial value problem. Such initial value solutions for the Eady and Charney problems (Farrell, 1984, 1982a, hereafter F2 and F1 respectively) reveal that the configurations identified by Petterssen (1955) as conducive to cyclogenesis give rise to large initial growth of nonmodal waves and efficiently excite normal modes whether these normal modes are unstable or not.

This excitation of neutral and decaying modes in the initial value problem suggests that an inconsistency in the normal mode instability approach to cyclogenesis may be overcome. The difficulty lies in the fact that normal-mode growth rates in the presence of Ekman damping are unrealistically small or negative (Card and Barcilon, 1982; Williams and Robinson, 1974). However, if the transient growth remains robust the mode may be excited to finite amplitude without its being exponentially unstable. To examine this possibility, solutions to the initial value problems of Eady and Charney in the presence of Ekman damping are examined.

2. Formulation of the Eady problem

The initial value problem for the model of Eady (1949) has the advantage of being solvable in terms of simple integrals (F2). Briefly, the shear and static stability are taken to be constant, the Boussinesq approximation made and the β effect ignored, resulting in a perturbation potential vorticity equation (Pedlosky, 1979):

$$\left(\frac{\partial}{\partial \tilde{t}} + i\tilde{\alpha}\tilde{z}\right)(\psi_{\tilde{z}\tilde{z}} - \tilde{\alpha}^2\psi) = 0 \tag{1}$$

where

$$\psi = \psi(\tilde{z}, \tilde{t})e^{i(\tilde{k}\tilde{x}+\tilde{l}\tilde{y})}$$

$$\tilde{\alpha} \equiv (\tilde{k}^2 + \tilde{l}^2)^{1/2}$$

and the following nondimensionalizations have been made:

$$\tilde{t} = \frac{t\Lambda\sqrt{\epsilon}\, k}{\alpha}$$

$$\tilde{k} = \frac{kH}{\sqrt{\epsilon}}$$

$$\tilde{z} = \frac{z}{H}$$

where $\epsilon = f^2/N^2$ is the square ratio of the Coriolis parameter to the Brunt-Väisälä frequency, H is the vertical scale and Λ the shear. The boundary conditions take into account the vertical velocity induced by Ekman suction (Pedlosky, 1979):

$$\frac{\partial^2\psi}{\partial\tilde{t}\partial\tilde{z}} - i\tilde{\alpha}(1 - \Gamma)\psi = 0, \quad \tilde{z} = 0 \qquad \text{(2a)}$$

$$\left(\frac{\partial}{\partial\tilde{t}} + i\tilde{\alpha}\tilde{z}\right)\psi_{\tilde{z}} - i\tilde{\alpha}(1 + \Gamma)\psi = 0, \quad \tilde{z} = \tilde{z}_T \qquad \text{(2b)}$$

where

$$\Gamma \equiv \frac{iN}{\Lambda H}\left(\frac{\nu}{2f}\right)^{1/2}\frac{\tilde{\alpha}^2}{\tilde{k}}$$

and ν is the vertical eddy viscosity coefficient.

Tildes are dropped in the sequel.

3. The damped Eady edge wave

The least complicated example of the excitation of a neutral mode in an initial value problem results from removing the upper boundary to infinity in this model and requiring only that the streamfunction be bounded as $z \to \infty$. The modal waves supported by the lower boundary are described in Gill (1982).

Noticing that the interior equation admits a particular solution for plane wave initial conditions as well as the bounded homogeneous solution, we write the general solution in the form:

$$\psi = \left[\frac{(1 + a^2)}{1 + (a - t)^2}\, e^{i(m-\alpha t)z} + A(t)e^{-\alpha z}\right]e^{i(kx+ly)}.$$

Here $a \equiv m/\alpha$. The boundary condition (2a) requires:

$$\frac{dA}{dt} + i(1 - \Gamma)A = -i(f - d)$$

with the convenient definitions:

$$f(t) \equiv \frac{2(1 + a^2)}{[1 + (a - t)^2]^2}$$

$$d(t) \equiv \frac{\Gamma(1 + a^2)}{1 + (a - t)^2}.$$

The solution subject to $A(0) = 0$ is

$$A(t) = -ie^{-i(1-\Gamma)t}\int_0^t e^{i(1-\Gamma)\tau}(f - d)d\tau.$$

While the amplitude of the modal component of the solution $|A|$ approaches a nonzero constant with time in the inviscid case, when $\Gamma \neq 0$ Ekman dissipation eventually damps it. Even so, the setup of the mode proceeds sufficiently rapidly to obtain an increase in amplitude for realistic parameter values. As an example we take $f = 10^{-4}$ s^{-1}, $N = 10^{-2}$ s^{-1}, $H = 10$ km, $\Lambda = 3$ m s^{-1} km^{-1}; confine the wave to a channel of width $L = 1500$ km in the meridional and choose $k = 3.0$ for a zonal wavelength of 2100 km. In the vertical $m = \pi$ corresponds to a westward tilt of π between $z = 0$ and $z = H$. The amplitude of the streamfunction at $z = 0$ for $\nu = 0$ and $\nu = 10$ m^2 s^{-1} as a function of nondimensional time $\tilde{t} = t\Lambda\sqrt{\epsilon}$ is shown in Fig. 1. Each unit of time corresponds to 9.3 h and we see that the growth of the wave over the first 18 h is robust in the presence of Ekman damping although the subsequent decay is quite rapid. This example suggests that cyclogenesis may be a rapid phenomena which does not depend upon the existence of exponential instability and which proceeds even when the modal solutions are strongly damped.

4. Eady initial value problem with damping

To better understand the relation between exponential instability and nonmodal transient growth in cyclogenesis, it is useful to solve the simplest problem which supports both. This is the model of Eady which results from placing a lid at $z = H$ and seeking a solution of (1) and (2) in the form (F2):

$$\psi = \left\{\frac{(1 + a^2)}{1 + (a - t)^2}\, e^{i(m-\alpha t)z} + A(t)\cosh(\alpha z) + B(t)\sinh(\alpha z)\right\}e^{i(kx+ly)}.$$

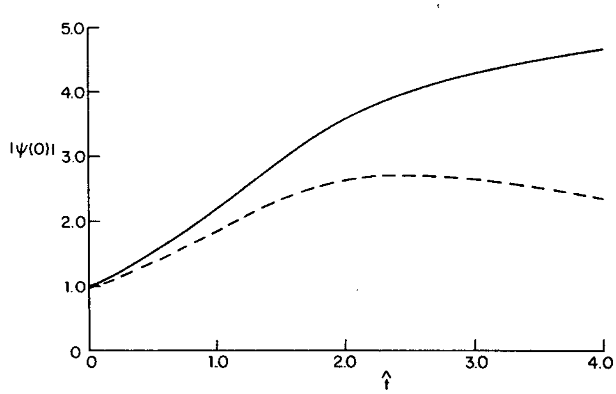


FIG. 1. Streamfunction amplitude at $z = 0$ as a function of nondimensional time for the Eady edge wave with $\nu = 0.0$ (solid) and $\nu = 10$ m^2 s^{-1} (dashed). The wave is confined to a meridional channel of width $L = 1.5$ and has zonal wavenumber $k = 3.0$ and a vertical wavenumber $m = \pi$ at $t = 0$ which corresponds to an energy releasing perturbation.

Boundary conditions (2) require:

$$\frac{dB}{dt} - i(1-\Gamma)A = i(f-d)$$

$$\frac{dA}{dt} + \coth(\alpha)\frac{dB}{dt} + i[\alpha - (1+\Gamma)\coth(\alpha)]A + i[\alpha\coth(\alpha) - (1+\Gamma)]B = \frac{ie^{i\alpha(a-t)}}{\sinh(\alpha)}(f+d)$$

The solution for $A(0) = B(0) = 0$ is

$$\left.\begin{aligned} A &= \frac{1}{\omega_1-\omega_2}[e^{\omega_1 t}L_1(t) - e^{\omega_2 t}L_2(t)] \\ B &= \frac{i}{\omega_1-\omega_2}\left[\frac{e^{\omega_1 t}}{\sigma_1}L_1(t) - \frac{e^{\omega_2 t}}{\sigma_2}L_2(t)\right] \end{aligned}\right\}$$

where use is made of the normal mode eigenvalues (Barcilon, 1964):

$$\omega_2^1 = i\left(-\frac{\alpha}{2} + \Gamma\coth\alpha\right) \pm \left(\alpha\coth(\alpha) - 1 - \frac{\alpha^2}{4} - \frac{\Gamma^2}{\sinh^2(\alpha)}\right)^{1/2} \quad (3)$$

and the definitions:

$$\left.\begin{aligned} L_1 &\equiv g_{-1}\left[-i\omega_1\coth(\alpha) - \frac{\omega_1\omega_2}{1-\Gamma}\right] + \frac{i\omega_1 e^{im}}{\sinh(\alpha)}g_{+2} \\ L_2 &\equiv g_{-2}\left[-i\omega_2\coth(\alpha) - \frac{\omega_1\omega_2}{1-\Gamma}\right] + \frac{i\omega_2 e^{im}}{\sinh(\alpha)}g_{+1} \\ g_{+j} &\equiv \int_0^t (f+d)e^{[\omega_j - 2i\Gamma\coth(\alpha)]\tau}d\tau \\ g_{-j} &\equiv \int_0^t (f-d)e^{-\omega_j\tau}d\tau \end{aligned}\right\}$$

5. Cyclogenesis in the Eady model with damping

Anticipating that rapid development will result when a surface depression is overtaken by an upper level short wave (Petterssen, 1955; Petterssen and Smebye, 1971), we choose a disturbance of the form:

$$\psi = h(x)e^{i(kx+mz+\pi/2)}\sin\left(\frac{\pi y}{L}\right) \quad (4)$$

with localizing function:

$$h(x) = \begin{cases} \dfrac{1-\cos(4\pi x/d)}{2}, & 0 \leqslant x < \dfrac{d}{4} \\ 1, & \dfrac{d}{4} \leqslant x \leqslant \dfrac{3d}{4} \\ \dfrac{1-\cos[4\pi(x-d/2)/d^{-1}]}{2}, & \dfrac{3d}{4} < x \leqslant d, \end{cases} \quad (5)$$

in a periodic zonal domain of length $D = 6\pi$. As in the previous example, channel width $L = 1.5$ and initial disturbance wavenumber $k_0 = 3.0$ corresponds to perturbation zonal wavelength $\lambda = 2100$ km. Initial vertical wavenumber $m = \pi$ is chosen to model an upper-level low one-half wavelength upstream of its lower-level counterpart. The perturbation is localized by taking $d = 2\lambda$ in (5) and a total of 128 wavenumbers are included in the Fourier representation of (4). The real part of ψ is shown in Fig. 2 and Fig. 3 for $\hat{t} = 0$, $\hat{t} = 1.0$, $\hat{t} = 2.0$ and $\hat{t} = 4.0$ corresponding to dimensional times of 9.3, 18.6 and 37.2 h. The undamped case $\nu = 0.0$ (Fig. 2) reproduces previous results (F2) while the $\nu = 10$ m^2 s^{-1} case (Fig. 3) establishes that transient growth gives rise to robust cyclogenesis even when the normal modes are damped. In this instance the least damped mode, at $k = 0.73$ has $1/e$ damping time of 298 h but plays an insignificant role in the excited spectrum. More to the point is the damping time of the central wavenumber $k = 3.0$ which is 41.3 h.

6. Cyclogenesis in the Charney problem with damping

While the Eady model enjoys the advantage of being more analytically tractable than the other canonical example of baroclinic instability, that of Charney (1947), it ignores the important physics of density stratification and the first order sphericity effect which is the variation of the Coriolis parameter. In addition, the tropopause is treated as a rigid boundary which supports its own Ekman layer, an approximation of questionable validity and one which forces an objectionable symmetry in the solution. In an attempt to overcome these limitations, the initial value problem for the Charney model with Ekman damping is examined.

We take an exponential density stratification $P = P_0 e^{-z/H}$ which provides the vertical scale H and retain the linear term in the variation of the Coriolis parameter $f = f_0 + \beta y$. The nondimensional perturbation potential vorticity equation for the scaled streamfunction:

$$\Phi = \psi(\tilde{z}, \tilde{t})e^{\tilde{z}/2}e^{i(\tilde{k}\tilde{x}+\tilde{l}\tilde{y})}$$

is (Pedlosky, 1979):

$$\left(\frac{\partial}{\partial\tilde{t}} + i\tilde{k}\tilde{z}\right)\left[\psi_{\tilde{z}\tilde{z}} - \left(\tilde{\alpha}^2 + \frac{1}{4}\right)\right]\psi + i\tilde{k}(\tilde{\beta}+1)\psi = 0 \quad (6)$$

with associated boundary conditions:

$$\frac{\partial}{\partial\tilde{t}}\left(\psi_{\tilde{z}} + \frac{\psi}{2}\right) - i\tilde{k}(1-\Gamma)\psi = 0, \quad \tilde{z} = 0 \quad (7a)$$

$$\psi \text{ bounded}, \quad \tilde{z} \to \infty \quad (7b)$$

where $\tilde{t} = t\Lambda\sqrt{\epsilon}$, $\tilde{\beta} = \beta H/\Lambda\epsilon$, and other quantities are the same as in the Eady example.

The solution of (6) and (7) for normal modes ψ

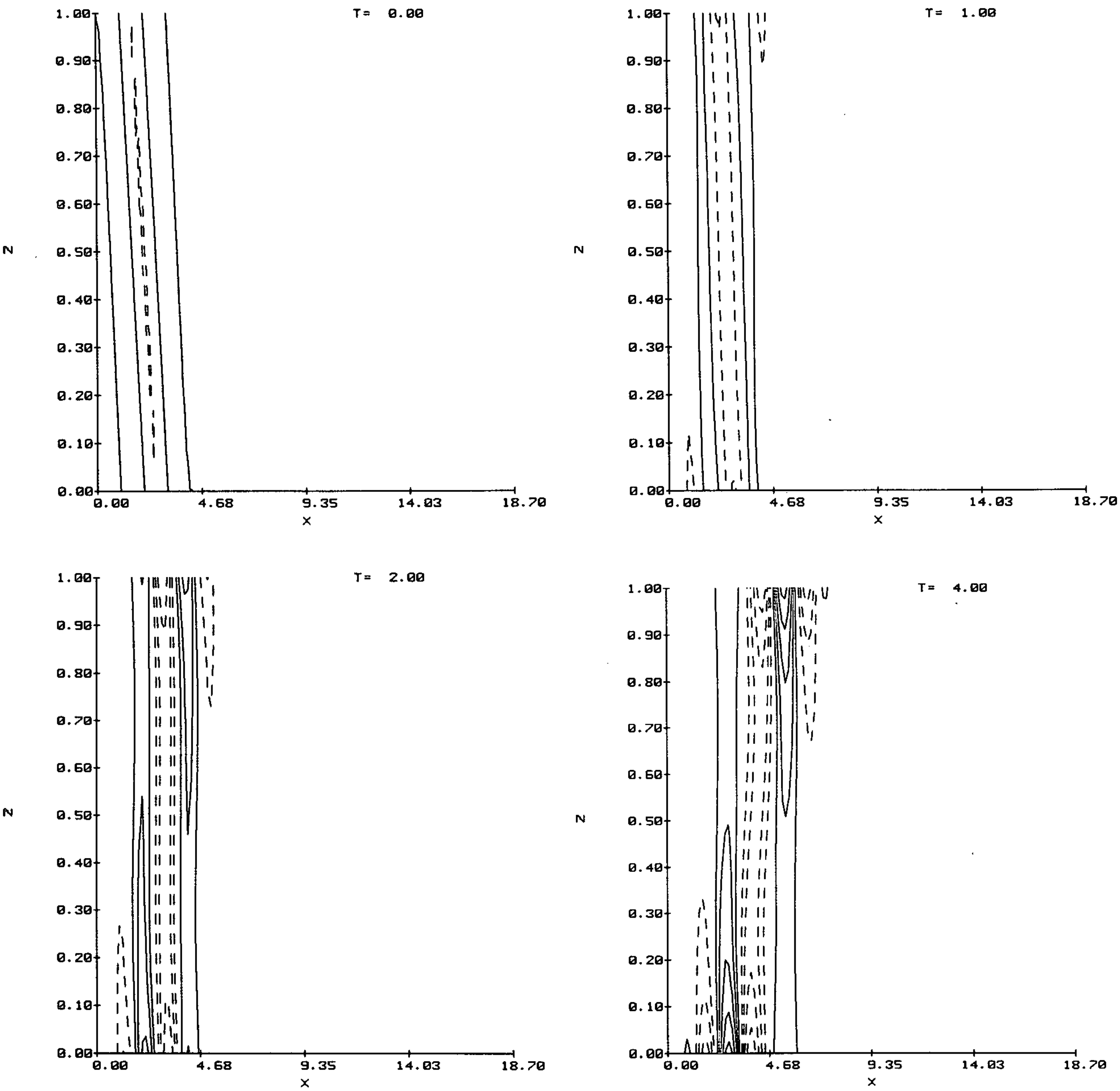


FIG. 2. Evolution in the undamped ($\nu = 0$) Eady model of an initial condition streamfunction which favors cyclogenesis. In (a) taken at $t = 0$, the perturbation is outlined by the zero contour which is suppressed in (b) at $t = 1.0$, (c) at $t = 2.0$ and (d) at $t = 4.0$. Negative contours are dashed and the contour interval is 1.0.

$= \psi(\tilde{z})e^{\sigma t}$ is well-known in the inviscid case (Charney, 1947; Kuo, 1979). It involves transforming (6) to a confluent hypergeometric equation in standard form and applying (7) to obtain the dispersion relation. Including Ekman damping does not increase the difficulty of the problem as (7) must be solved iteratively in any case (Card and Barcilon, 1982).

The inviscid Charney problem is found to be unstable at all but a few isolated zonal wavenumbers for shears typical of midlatitudes. This has encouraged the view that the westerlies as observed are unstable to exponential perturbations. However, including Ekman damping sharply reduces both the strength of the instability and the region of wavenumber space occupied by unstable waves. For example, taking $f = 10^{-4}$ s^{-1}, $N = 10^{-2}$ s^{-1}, $H = 10$ km, $\Lambda = 3$ m s^{-1} km^{-1}, $\beta = 1.6 \times 10^{-11}$ m^{-1} s^{-1} gives $\tilde{\beta} = 0.53$ for the nondimensional parameter characterizing the Charney problem. Taking $l = 2.0$, corresponding to a wavelength of 3100 km, we find the dispersion relation for $\nu = 0.0$ shown in

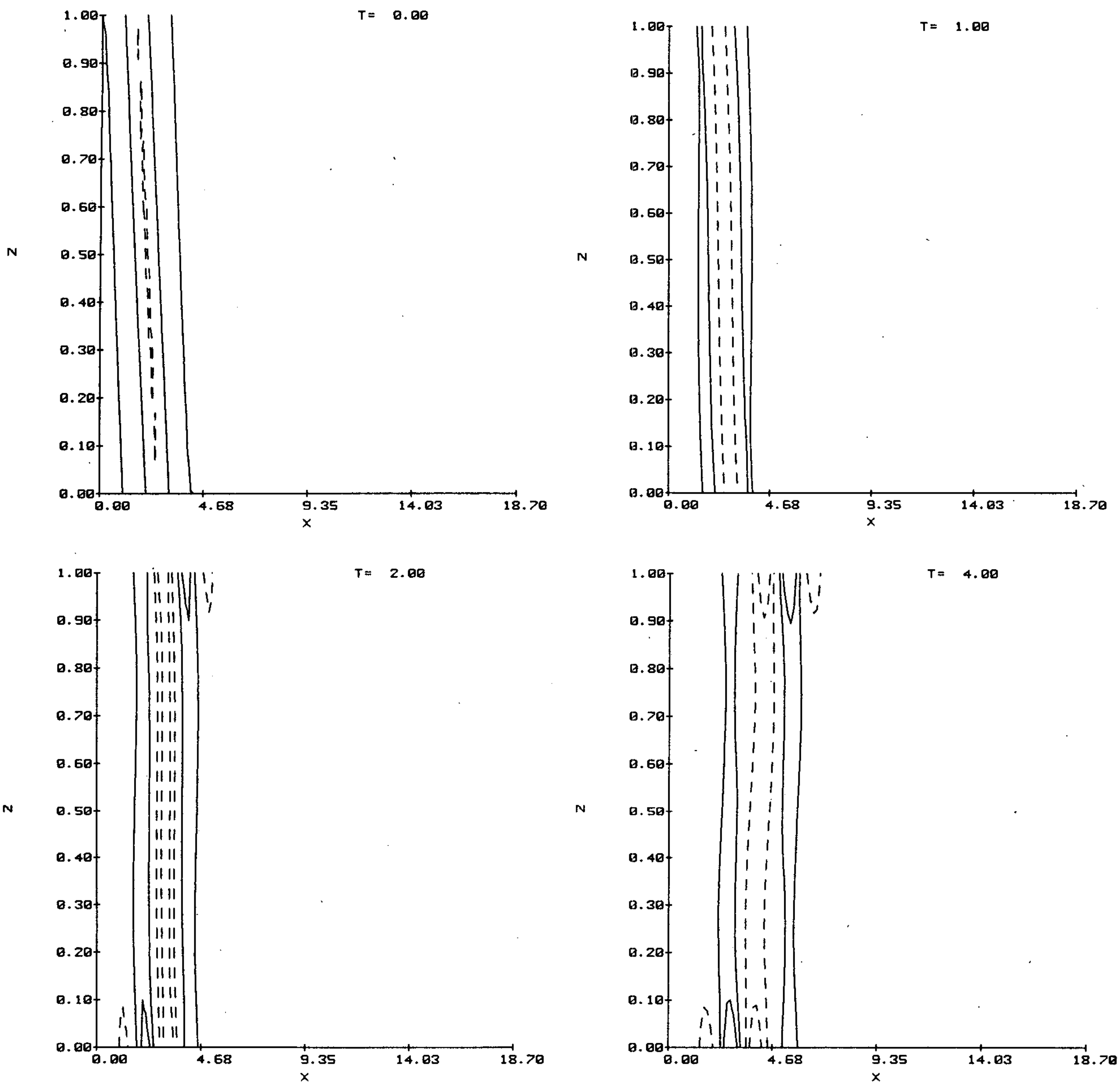


FIG. 3. As in Fig. 2 except Ekman damping corresponding to a vertical diffusion $\nu = 10\ \mathrm{m^2\ s^{-1}}$ is included.

Fig. 4. Maximum growth rate occurs at $k = 2.2$ and gives *e*-folding time of 67 h. Increasing ν to $2.5\ \mathrm{m^2\ s^{-1}}$ results in the second dispersion relation plotted. Maximum growth for this damping occurs at $k = 1.6$ and gives 243 h for an *e* fold. Attempts to raise the value of ν to $5\ \mathrm{m^2\ s^{-1}}$ resulted in a complete loss of instability. In fact, as shown in Fig. 5, the growth rate is a strong function of

$$\Gamma_0 \equiv \frac{N}{\Lambda H}\left(\frac{\nu}{2f}\right)^{1/2},$$

the nondimensional measure of the spindown, and stabilization results for $\Gamma_0 = 0.05$ corresponding to $\nu = 4.5\ \mathrm{m^2\ s^{-1}}$.

The Eady model supports two normal modes at each zonal wavenumber both in the damped and undamped case as can be seen from the dispersion relation (3). By contrast, while the Charney mode example in Fig. 4 supports an exponentially decaying mode for all k, the growing mode does not exist for values of k which exceed the short wave cutoff at $k \approx 3.2$ (Card and Barcilon, 1982). For the square wave $k = l = 2.0$, Fig. 5

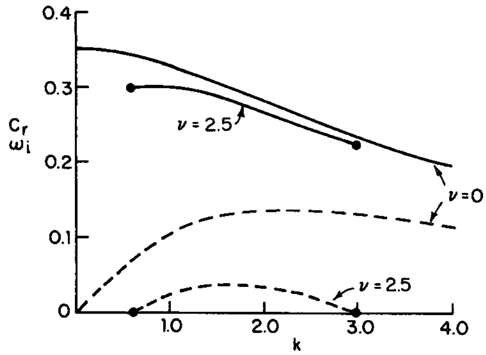


FIG. 4. Dispersion relation for the Charney model with damping compared to the inviscid dispersion relation. Phase speed c_r (solid) and imaginary part of frequency ω_i (dashed) are shown for $\beta = 0.53$ and $l = 2.0$ which corresponds to a wavelength of 3100 km and typical midlatitude parameter values.

shows that the mode is lost for the equivalent of $\nu \geqslant 4.5$ m^2 s^{-1}. Analysis reveals that the growing normal mode acquires a singularity as its growth rate goes to zero and that it joins the continuous spectrum of singular neutral modes. For values of $\nu \geqslant 4.5$ m^2 s^{-1} in our example this leaves only the highly decaying continuation of the inviscid stable mode to support disturbances of modal form. However, the extreme eastward tilt with height of this mode and its rapid decay in time results in its playing a negligible role in the spectrum of deepening perturbation. Thus linear development of synoptic scale waves in the presence of moderate dissipation results almost entirely from the continuous spectrum.

At this point an additional comment on the existence of unstable normal modes is appropriate. In a zonally varying basic flow, such as the atmosphere, physically relevant unstable normal modes are coincident with absolute instabilities (Merkine, 1977; Farrell, 1982b; Pierrehumbert, 1984). A rule of thumb for the existence of absolute instability is that the group velocity calculated using the real part of the dispersion relation $c_g = c_r + k(\partial c_r/\partial k)$ be negative in the unstable region. Examining Fig. 4 we see that the region of negative c_g is eliminated by friction. Other examples not shown here reveal this to be a quite general result and suggests that absolute instability is confined to regions of zero or negative mean velocity (Lindzen *et al.*, 1983).

Returning to the Charney model, the initial value integration can be done using any of a number of methods including that described in F1. Briefly, we discretize (6) in the vertical and apply the finite difference analogue of (7) to obtain a matrix representation of the system which can then be solved for its eigenvectors. The solution at later time is found by summing over the eigenvectors, taking into account the time dependence of each. The advantage of this method is that the approximate eigenmodes and eigenvalues are obtained as well as the finite difference approximations to the continuous spectrum modes.

The Charney model was integrated for the initial condition:

$$\Phi = h(z)e^{i(kx+ly+(3\pi/4)z)}$$

$$h(z) = \left[1 - \tanh\left(\frac{z-1.25}{0.25}\right)\right]\Big/2.0$$

with parameter values as in the previous example ($f = 10^{-4}$ s^{-1}, $N = 10^{-2}$ s^{-1}, $H = 10$ km, $\Lambda = 3$ m s^{-1} km^{-1}, $\beta = 1.6 \times 10^{-11}$ m^{-1} s^{-1}) and wavenumbers $k = l = 2.0$ corresponding to a wavelength of 3100 km. A rigid boundary placed at $z = 4.0$ for computational convenience was found not to affect the solution.

Results are presented for the undamped case (Fig. 6) and with damping corresponding to $\nu = 10$ m^2 s^{-1} (Fig. 7).

The cyclone obtains greater depth in the inviscid case, but even with the damping resulting from $\nu = 10$ m^2 s^{-1} a robust cyclogenesis results. The total-energy growth rate plots (Fig. 8a, b) show the inviscid case obtaining an asymptotic growth characteristic of the exponential normal mode while the damped case loses energy after obtaining its maximum transient growth. Recall that in the latter case there are no normal modes with the exception of the negligibly excited continuation of the exponential decaying inviscid normal mode.

7. Discussion and conclusions

Exponential instability of baroclinic waves at synoptic cyclone scale is insignificant or nonexistent when Ekman damping corresponding to moderate values of vertical diffusion is included. The observed transient deepening of disturbances is here alternatively ascribed to the release of mean flow potential energy by favorably configured perturbations of nonmodal form.

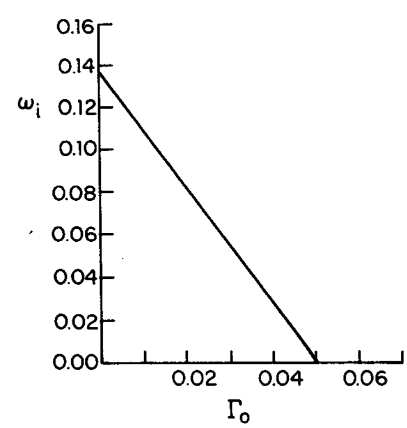
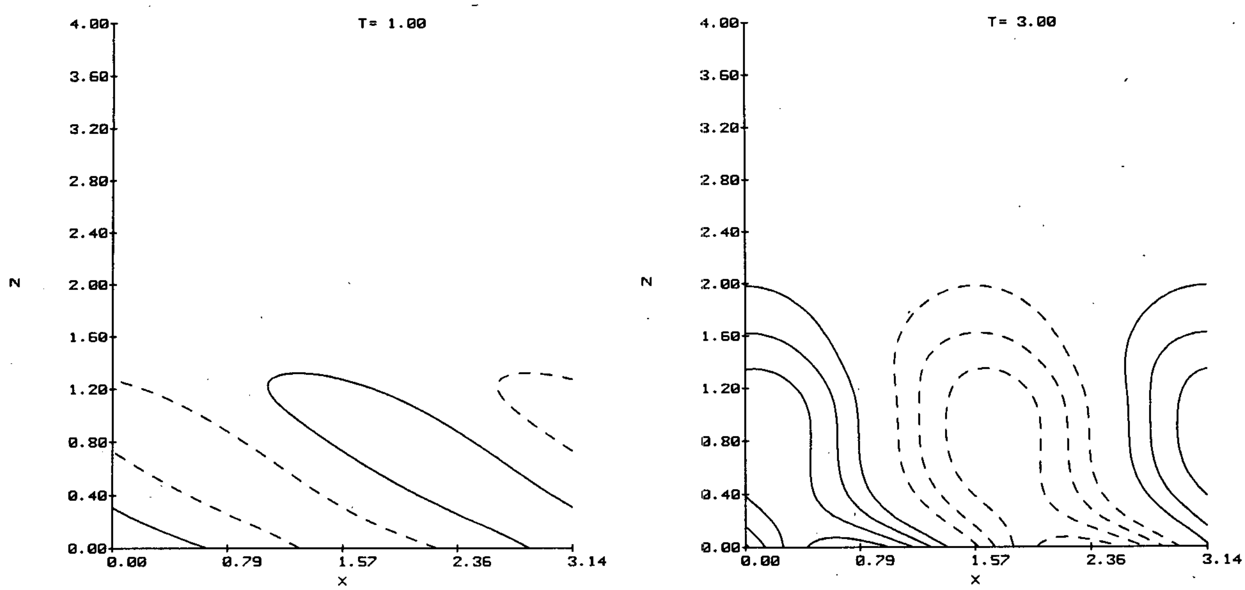


FIG. 5. Maximum growth rate of the Charney mode as a function of the Ekman parameter. Other parameter values are as in Fig. 4 except $k = l = 2.0$, $\Gamma_0 = 0.05$ corresponds to $\nu = 4.5$ m^2 s^{-1} for these values.



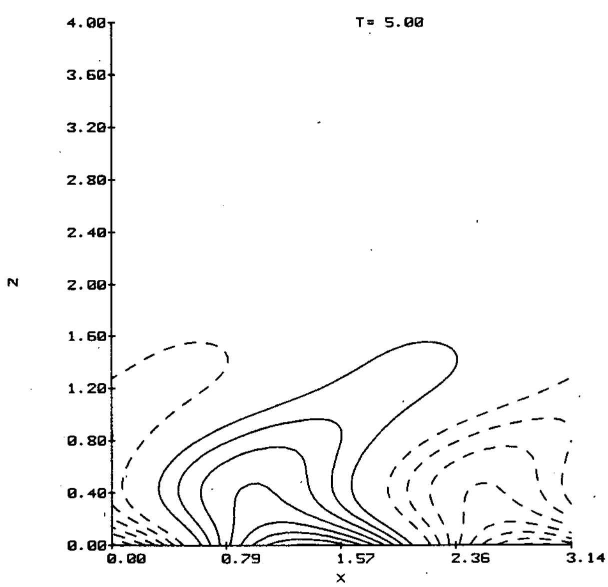


FIG. 6. Evolution in the undamped Charney model of an initial condition which favors cyclogenesis. Nondimensional times of (a) $t = 1.0$, (b) $t = 3.0$, (c) $t = 5.0$ correspond to 9.3, 27.9 and 46.5 h, respectively. The contour interval is 1.0, dashed contours are negative and the zero contour has been suppressed.

Evidence for initial conditions of finite amplitude leading to cyclogenesis is widespread in synoptic meteorology. Forecast rules developed for midlatitude cyclones are explicit in requiring the configuration of low level perturbation and upper level trough for development which we have modeled here (Petterssen, 1955; Palmen and Newton, 1969; chapter 11). Similar configurations have been implicated in the formation of monsoon depressions (Koteswaram and George, 1958) and may explain the association of these lows with propagating upper level disturbances (Saha *et al.*, 1981).

Viewing these ideas in a wider context, we note that the midlatitude atmospheric structure is such that synoptic scale baroclinic waves are found to lie near their neutral curve in parameter space. The baroclinic adjustment hypothesis (Stone, 1978; Lindzen and Farrell, 1980) which has been advanced to explain the maintenance of the midlatitude temperature gradient requires the gradient to rise under radiative driving until the stability boundary is reached. If that boundary is determined by friction, which presumably it must be for $\Gamma_0 = 0$ in the Charney model requires a zero gradient, then the climate must be greatly influenced by those parameters which control Ekman dissipation including

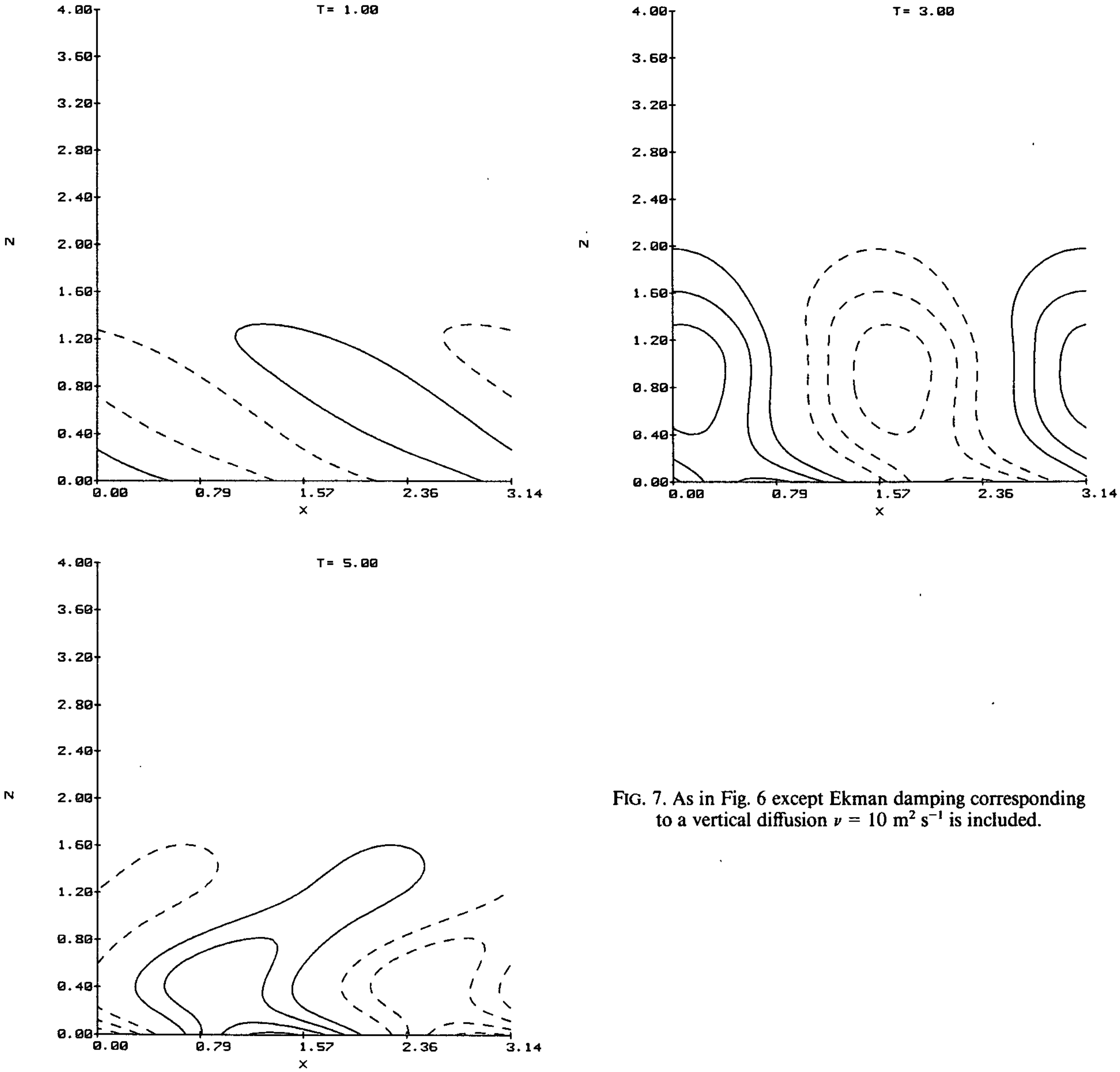


FIG. 7. As in Fig. 6 except Ekman damping corresponding to a vertical diffusion $\nu = 10\ m^2\ s^{-1}$ is included.

the surface roughness. However, in light of the foregoing results a second possibility must be entertained: that the relaxation of the temperature gradient arises from the growth of perturbations on a stable mean flow. If this release of potential energy by perturbations is robust enough, the system may be maintained on the stable side of the exponential instability boundary. This mechanism places the sources of energy-releasing disturbances in the atmosphere in the role of determining the temperature gradient and suggests that different climates may arise depending on those sources.

The relaxation of the temperature gradient through the growth of these eddies does not depend on the degree of instability of the flow. To the contrary, eddy fluxes are associated with a reduction of the temperature gradient and thereby the instability because the fluxes peak when the sources are active. Such a negative correlation between synoptic scale wave activity and temperature gradients has in fact been found (Lorenz, 1979; Stone *et al.*, 1982; Ghan, 1984).

Sources of energy releasing perturbations presumably include internal heating and boundary interactions. If, for instance, a GCM were to lack sufficient physics to model accurately the generation of distur-

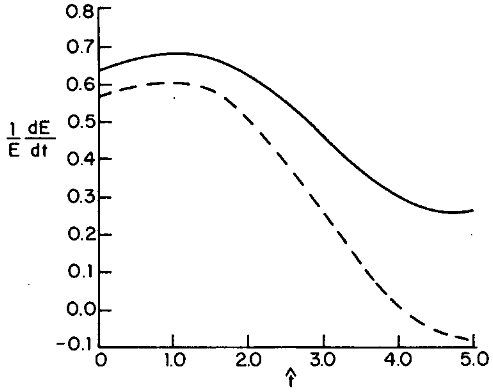


FIG. 8. The total-energy growth rate for the example in Fig. 6, with $\nu = 0$ (solid), and Fig. 7, $\nu = 10\ m^2\ s^{-1}$ (dashed).

bances, then it would spin up in zonal velocity until it exceeded the actual stability boundary which, for realistic Ekman damping, may be far from observed climatology.

Integral constraints allow quite general arguments to be made concerning the structure and evolution of nonmodal disturbances which develop in shear flows. The analyses of Pierrehumbert (1983) and Zeng (1983) are particularly useful in cases where the contribution of boundaries and normal modes can be neglected. Note, however, that the presence of boundaries and normal modes can qualitatively effect the results (Farrell, 1982).

Additional remarks follow:

- Ekman damping is not the only way a baroclinic flow may be stabilized to linear perturbations. If the gradient of potential vorticity is everywhere positive and there is no temperature gradient at the surface, the necessary condition for instability is violated (Charney and Stern, 1962). Even in this case the initial value problem produces robust growth if there is available potential energy in the basic flow (Farrell, 1982a).
- The common device of introducing a Rayleigh friction has been shown to produce much weaker dissipation than that resulting from Ekman damping. Hendon and Hartman (1982) find a Rayleigh friction corresponding to a 10 day spindown to be equivalent to $\nu = 0.1\ m^2\ s^{-1}$ in their planetary wave model. This indicates that realistic models of synoptic and planetary atmospheric waves must take account of the primary dissipative mechanism which is Ekman damping.
- Some instances of cyclogenesis may be associated with the downstream development of baroclinic Rossby waves traveling as a time dependent packet. Such waves are of modal form when account is taken of their complex frequency and wavenumber and they are associated with local baroclinic energetics (Merkine, 1977; Simmons and Hoskins, 1979; Farrell, 1982b). For the assumption of a finite amplitude initial disturbance, these waves would not require large growth rates to produce a series of cyclogenesis events and therefore they may be compatible with the long development times implied for damped waves.
- It is possible that in regions of intense shear, low static stability and reduced vertical diffusion the growth rate of unstable modes may be sufficiently rapid to account for cyclogenesis. These events would correspond to type A development in the nomenclature of Petterssen and Smebye (1971) while the mechanism of this paper corresponds to their type B. Likely regions for normal mode development may include fronts such as the North American East coast front and cold fronts associated with intense cyclones. Note, however, that such modal waves may themselves become involved in a type B development if they come under the influence of an upper level vorticity center.

It is hoped that by providing a conceptual framework for distinguishing between types of development this work will stimulate observations resulting in a better understanding of the process of cyclogenesis.

Acknowledgement. This work was supported by NASA grant NGL 22-007-228.

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