Transient Development in Confluent and Diffluent Flow

BRIAN F. FARRELL

Department of Earth and Planetary Sciences, Harvard University, Cambridge, Massachusetts

(Manuscript received 12 January 1988, in final form 28 April 1989)

ABSTRACT

Explaining the growth of disturbances superimposed on mean flows is a central problem in meteorology. Most widely studied models of the development process involve perturbations to shear flows with shear restricted to the meridional direction. Recently the importance of zonal variation of the mean flow was recognized and the study of shear flows extended to include zonal variation in shear. These studies found that the eigenfunctions associated with unstable modes in the simple shear problem are highly sensitive to zonal variation of the mean flow. However, there also exists another mechanism for development in a zonally inhomogeneous flow field: transient growth not associated with exponential instability. Properly configured perturbations exhibit transient growth in deformation fields associated with regions of confluence and diffluence at rates comparable to development in shear flow.

In this work analytic solution of the linear initial value problem for the barotropic vorticity equation in deformation flow is used to construct local perturbations that undergo rapid transient development. Implications for cyclogenesis and block formation are discussed.

1. Introduction

The atmosphere exhibits energetic interactions with scales from the viscous subrange to the planetary. While it is generally accepted that the Navier–Stokes equations are an adequate framework for the description of fluid dynamics at all these scales, the complexity of the complete equations often prevents their direct application to the interpretation of phenomena. Two of the most powerful techniques to overcome this difficulty are the use of simplified equations and linearization about a suitably defined background flow. Application of these techniques to the origin and growth of traveling waves at synoptic scale in the atmosphere has traditionally employed such approximate equations as the barotropic vorticity or quasi-geostrophic and assumed a streamwise homogeneous shear flow as the background around which to linearize (Charney 1947; Eady 1949; Kuo 1949). The familiar eigenproblem that arises focuses attention on the structure and growth rate of exponential normal modes and on conditions that the background flow must satisfy in order to support these exponentially growing modes. Despite the near universal acceptance of this exponential normal mode paradigm for development in shear it was recognized early that the assumption of this solution form for the wave was restrictive and that there existed another class of perturbations that could affect stability (Thomson 1887; Orr 1907). These early investigators showed by example that for properly configured initial conditions rapid growth of perturbation energy and amplitude could occur without exponentially unstable normal modes. For the quasi-geostrophic equations appropriate to the cyclone problem in baroclinic shear, Farrell (1982, 1984, 1985) found that allowing for combinations of unstable and stable (but not orthogonal and perhaps singular) modes expanded the set of allowable perturbations to include members with structure and growth characteristics in better agreement with observations of cyclogenesis.

Both the instantaneous and time-mean structure of the midlatitude winds are dominated by concentration of the flow into jet streams sharply confined in latitude and height. Because of its continual variation in space and time, however, a jet stream is not well represented by a zonal mean and it is important to account for the influence of zonal variation of the jet on the stability problem. Eigenvalue calculations on zonally inhomogeneous flows include work of Niehaus (1980), Frederiksen (1979), Merkine and Balgovind (1983), Pierrehumbert (1984), Simmons et al. (1983), Branstator (1985), and Fyfe and Derome (1987). These and the observational analysis of Hoskins et al. (1983) have shown that in addition to modification of the energetics associated with eigenfunctions of zonally homogeneous flow additional energetic interactions between the mean and the perturbations arise due to the zonal inhomogeneity itself. Such interactions are to be expected from general energy integral relations (Pierrehumbert 1983) and some illustrative solutions

Corresponding author address: Brian F. Farrell, Department of Earth and Planetary Sciences, Harvard University, Pierce Hall, 29 Oxford Street, Cambridge, MA 02138.

exist (Craik and Criminale 1986; Bretherton and Haidvogel 1976) but their role in the atmosphere has not been widely explored.

Here a simple model that isolates the development of perturbations in regions of confluence and diffluence is examined and an assessment of the importance of this interaction in the atmosphere is made.

2. The model

Consider an irrotational and nondivergent velocity field with streamfunction:

$$\Psi = \alpha xy. \tag{2.1}$$

Velocities in the zonal and meridional directions are $U = -\alpha x$, $V = \alpha y$. If variation of the Coriolis parameter is ignored this is a stationary solution of the unbounded barotropic vorticity equation, otherwise an external forcing field that does not affect the linearized problem is required to maintain it. Although it does not satisfy commonly imposed boundary conditions such as vanishing normal velocity on plane parallel surfaces, the deformation field is a good approximation to the local flow in regions of confluence and diffluence (Bretherton and Haidvogel 1976; Shutts 1983). Deformation fields are characteristic of the local time mean upper level winds in the entrance and exit regions of storm tracks as well as of less permanent regimes in other regions, e.g., western Europe.

The linearized nondivergent barotropic vorticity equation is:

$$\frac{\partial \nabla^2 \psi}{\partial t} + \left[-(\mathrm{sgn}\alpha)x\frac{\partial}{\partial x} + (\mathrm{sgn}\alpha)y\frac{\partial}{\partial y}\right]\nabla^2\psi + \frac{\partial \psi}{\partial x}\beta = \upsilon\nabla^2(\nabla^2\psi), \tag{2.2}$$

in which time has been nondimensionalized by α^{-1} and distance by a typical synoptic length scale $L = 500$ km so that the problem is characterized by a nondimensional beta parameter, $\beta = \beta_{\mathrm{dim}}(L/\alpha)$ and $\upsilon = V_{\mathrm{dim}}/L^2\alpha$, an inverse Reynolds number. Appropriate values for α range from 1×10^{-5} s^{-1} in the entrance and exit regions of the climatological storm tracks to 5×10^{-5} s^{-1} locally in regions of strong diffluence (Shapiro 1976). Choosing a midlatitude value of $\beta_{\mathrm{dim}} = 1.6 \times 10^{-11}$ s^{-1} m^{-1} and $\alpha = 1 \times 10^{-5}$ s^{-1}, corresponding to 27.8 h for a unit of nondimensional time, gives $\beta = 0.8$. Taking $\upsilon = 1$ requires $\upsilon_{\mathrm{dim}} = 2.5 \times 10^6$ m^2 s^{-1}, a large value that justifies use of inviscid dynamics for integration over a restricted time interval of sufficiently smooth initial conditions.

Time scales associated with horizontal confluence and diffluence are comparable to vertical shear time scales associated with baroclinic development when account is taken of the scaling of vertical shear by the ratio of the Coriolis parameter to the Brunt–Väisälä frequency, $f/N \approx 10^{-2}$ in the baroclinic problem. A typical vertical shear is 3×10^{-3} s^{-1}, giving dimensionally 9.3 h for one unit of nondimensional time. Considering that perturbation growth takes place on the time scale of the strain rate, growth rates for the most favorably configured disturbances interacting with deformation fields are likely to be of the same order as baroclinic growth rates. Unfortunately, the lack of exponential normal mode instabilities in deformation fields has inhibited acceptance of the role of these regions in development. The examples to follow demonstrate that properly configured perturbations in these flows exhibit growth comparable to that found in baroclinic shear.

Consider first the solution for a single plane wave of infinite extent with streamfunction:

$$\psi = F(t)e^{[i(k(t)x + l(t)y) + D(t)]}.$$

With the choice $\alpha > 0$ kinematics requires the zonal wavenumber increase while the meridional wavenumber decreases:

$$k(t) = k_0 e^t \tag{2.3a}$$

$$l(t) = l_0 e^{-t}. \tag{2.3b}$$

Viscosity produces a damping proportional to the square of the instantaneous wavenumber:

$$D(t) = -\upsilon \int_0^t (k(\tau)^2 + l(\tau)^2)d\tau.$$

This can be reduced using (2.3) to

$$D(t) = -\upsilon\left[\frac{k_0^2}{2}(e^{2t} - 1) + \frac{l_0^2}{2}(1 - e^{-2t})\right].$$

The complex amplitude factor includes a time dependent phase due to the β effect:

$$F(t) = F_0 \frac{k_0^2 + l_0^2}{k(t)^2 + l(t)^2} e^{i\beta \int_0^t k(\tau)/k(\tau)^2 + l(\tau)^2 d\tau}. \tag{2.4}$$

It is easily seen that wave energy varies by the same factor as the amplitude:

$$E(t) = E_0 \frac{k_0^2 + l_0^2}{k(t)^2 + l(t)^2}. \tag{2.5}$$

Integral inequalities can be used to bound the growth of perturbation energy by the mean flow strain rates (Pierrehumbert 1983). In the familiar case of inviscid parallel flow with $U = U(y)$ this bound is

$$\frac{d\ln E}{dt} \leqslant \left|\frac{dU}{dy}\right|_{\max},$$

while for the deformation (2.1) it is

$$\frac{d\ln E}{dt} \leqslant 2|\alpha|. \tag{2.6}$$

Experience shows these bounds typically allow for much more rapid growth than is found for unstable normal modes even for the wavenumber at which the exponential normal mode growth rate is maximized. Growth rates of normal modes at wavenumbers other than that of the maximum fall off rapidly, often vanishing for wavenumbers exceeding a short-wave cutoff. It is remarkable that the choice $k_0 = 0$, $l_0 \neq 0$ results from (2.5) in an inviscid growth at the integral bound maximum (2.7) and that this growth rate is attained for any l_0 wavenumber. Even though the growth rate is exponential the perturbation is not of normal mode form, being characterized by continual decrease in the wavenumber l.

An excitation often comprises a range of waves each of which can be characterized by its initial ratio of meridional to zonal wavenumber $\gamma_0 = l_0/k_0$. The amplification factor (2.4) attains a maximum

$$\frac{F_{\max}}{F_0} = \frac{1 + \gamma_0^2}{2\gamma_0} \qquad (2.7)$$

at the time which solves

$$\gamma_0 = e^{2t}. \qquad (2.8)$$

The wave at this time has an aspect ratio of 1 with both wavenumbers assuming the geometric mean of the initial wavenumbers $k = 1 = \sqrt{k_0 l_0}$. We anticipate that waves corresponding to greater initial values of γ_0 dominate as the wavenumber decreases with time resulting in an expansion of disturbance scale. Scale expansion is interrupted in this nondivergent model only by truncation of the initial disturbance spectrum and examples to follow show that perturbations in confluent and diffluent flows exhibit scale expansion rather than scale collapse as would be expected from the fate of a single wave.

The dominant wave at a particular time has undergone rapid growth having reached according to (2.7) evaluated at the time (2.8) the approximate amplitude

$$\frac{F(t)}{F_0} \approx \frac{e^{2t}}{2}.$$

Reference to (2.5) shows that this is also the energy growth for that wave and remarkably it is fully half the increase permitted by the energy integral bound (2.6). It is important to notice, however, that the expansion of scale is a phenomenon separate from the overall growth in perturbation energy that depends on the configuration of the initial disturbance. If all wavenumbers are equally excited the scale expansion occurs without an increase in the total energy integrated over all waves, as is also the case for such excitations in shear (Shepherd 1985).

Because the evolution of a perturbation in a confluent/diffluent flow is dependent on the form of the perturbation and has these two separable attributes of scale expansion and energy growth, it is worthwhile to examine the former in isolation by restricting the perturbation to have amplitude distributed isotropically in wavenumber (the plane wave examined above serves as an example of a perturbation growing in both scale and energy).

From the expression for wave amplitude (2.5), the energy of a plane wave is easily seen to evolve according to

$$E(k_0, l_0; t) = E_0(k_0, l_0)\frac{k_0^2 + l_0^2}{k_0^2 e^{2t} + l_0^2 e^{-2t}}.$$

It is appropriate for disturbances isotropically distributed in wavenumber to set $\mu_0^2 = k_0^2 + l_0^2$, so that $k_0 = \mu\cos\theta$ and $l_0 = \mu\sin\theta$ and the energy of the perturbation is

$$\bar{E} = \int_0^\infty \int_{-\pi}^{\pi} \frac{E_0(\mu, \theta)\mu d\mu\theta}{e^{2t}\cos^2\theta + e^{-2t}\sin^2\theta}. \qquad (2.9)$$

It suffices to consider the development of the excitation consisting of a circular delta function $E_0(\mu, \theta) = \hat{E}_0\delta(\mu - \mu_0)/(2\pi\mu)$ defined so that (2.9) reduces to

$$\bar{E} = \frac{\hat{E}_0}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta}{e^{2t}\cos^2\theta + e^{-2t}\sin^2\theta},$$

which is tabulated:

$$\bar{E} = \frac{\hat{E}_0}{2\pi}(\tan^{-1}[e^{-2t}\tan\theta|^{\pi}_{-\pi}]).$$

This integral has the value $\hat{E}_0 2\pi\mu_0$ for all t with contributions increasingly concentrated near $\theta = \pm(\pi/2)$ as time advances. This corresponds for $t \gg 1$ to a concentration of the constant energy into wavenumbers near zero, the increase in scale being clear from the decrease in wavenumber of the dominant Fourier components. The initial disturbance assumes the form of the Bessel function of order zero $\psi_0(r) = (2\hat{E}_0^{1/2}/\mu_0)J_0(\mu_0 r)$, $r = (x^2 + y^2)^{1/2}$ and an asymptotic solution as $t \rightarrow \infty$ can be found that is similar to that for the case of parallel shear flow (Farrell 1987).

Contrary to the plausible supposition that localized perturbations collapse in a deformation field, this result shows that disturbances exhibit monotone scale expansion at constant amplitude, a behavior limited in this nondivergent model only by the initial perturbation's supply of favorable wavenumbers. For localized disturbances this constant amplitude scale expansion regime intercepts the decay of a plane wave. The sequence of events predicted for the development of a favorably configured localized disturbance is for a period of rapid growth to be followed by a period of nearly constant amplitude scale expansion before the exhaustion of large γ_0 wave components in the spectrum of the original disturbances forces an ultimate decline. (This phenomena is evident in the model studied by Boyd and Christidis 1987.)

An example disturbance symmetric in wavenumber is chosen to isolate the scale expansion effect without introducing the distraction of an initial growth stage. The Gaussian initial perturbation,

$$\psi = -e^{-(x^2+y^2)}$$

and its temporal evolution are shown in Fig. 1.

The method employed to obtain this solution and those to follow is a Riemann sum of the Fourier transform

$$\psi(x, y, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{\psi}(k, l, t)e^{i(kx+ly)}dkdl \quad (2.10)$$

using waves evenly distributed over wavenumbers having appreciable amplitude. Time dependence of the component wave complex amplitudes are obtained from (2.4) and physical significance is ascribed to the real part of ψ. In this and subsequent figures the contour interval is adjusted to preserve the number of contours so that resolution is not compromised while maximum streamfunction magnitude and the total perturbation energy are indicated explicitly. The scale expansion effect is clearly revealed in Fig. 1.

Both rapid nonmodal development and scale expansion characterize the simple example of a plane wave, while the example of a symmetric excitation isolates the scale expansion effect. These ideas will be useful in interpretation of wave packet excitations to follow.

3. Development of localized asymmetric disturbances

Explosive transient development episodes are associated with the occurrence of favorably configured perturbations propagating into regions of confluence or diffluence. This kind of development, not related to instability or shear but nonmodal and transient in nature, has received little attention in the theoretical literature but is commonly seen in observations.

An isolated and zonally elongated initial disturbance entering a diffluent region is modeled as

$$\psi(x, y; t = 0) = -e^{-[(x+8/2)^2+(2y)^2]}. \quad (3.1)$$

Development of this perturbation is shown in Fig. 2. Dissipation and the β effect are ignored in this example. The dynamics displayed in Fig. 2 can be understood as a translation by the background flow, rapid transient growth due to interaction with the deformation, and an asymptotic scale expansion. These phenomena are illustrated by plotting the perturbation energy as a function of time for the perturbations introduced above: the plane wave with $l = 1.0$, $k = 0.0$; the plane

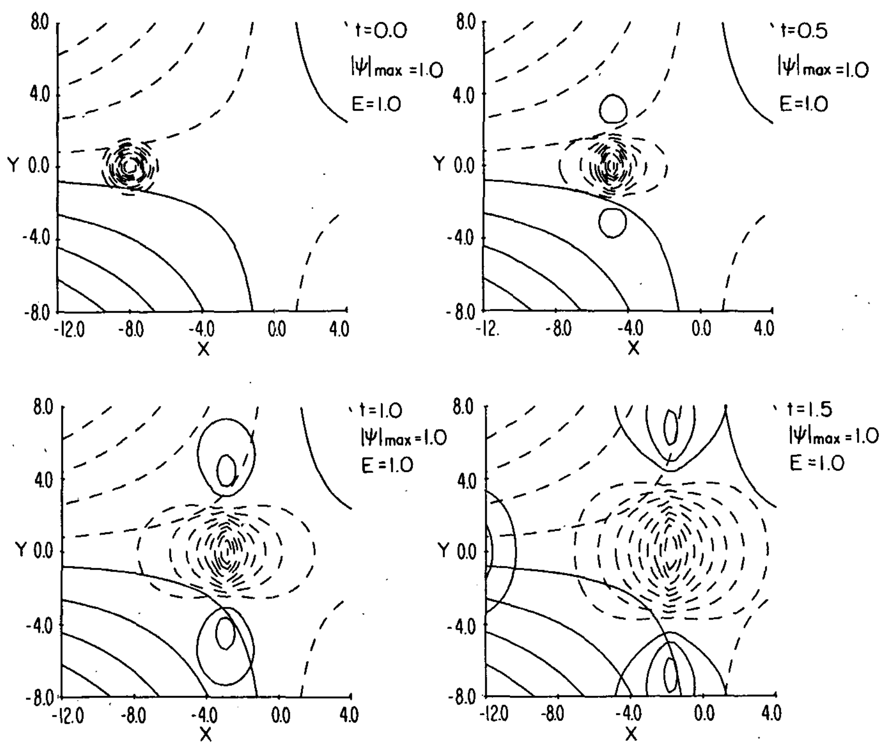

FIG. 1. Streamfunction of the symmetric Gaussian disturbance as a function of time (negative contours dashed). Maximum amplitude of the streamfunction and perturbation energy, both normalized by their respective values at $t = 0.0$ are indicated. The mean deformation field streamlines are also shown. The scale expansion effect is clearly revealed but there is no increase in perturbation energy or amplitude.

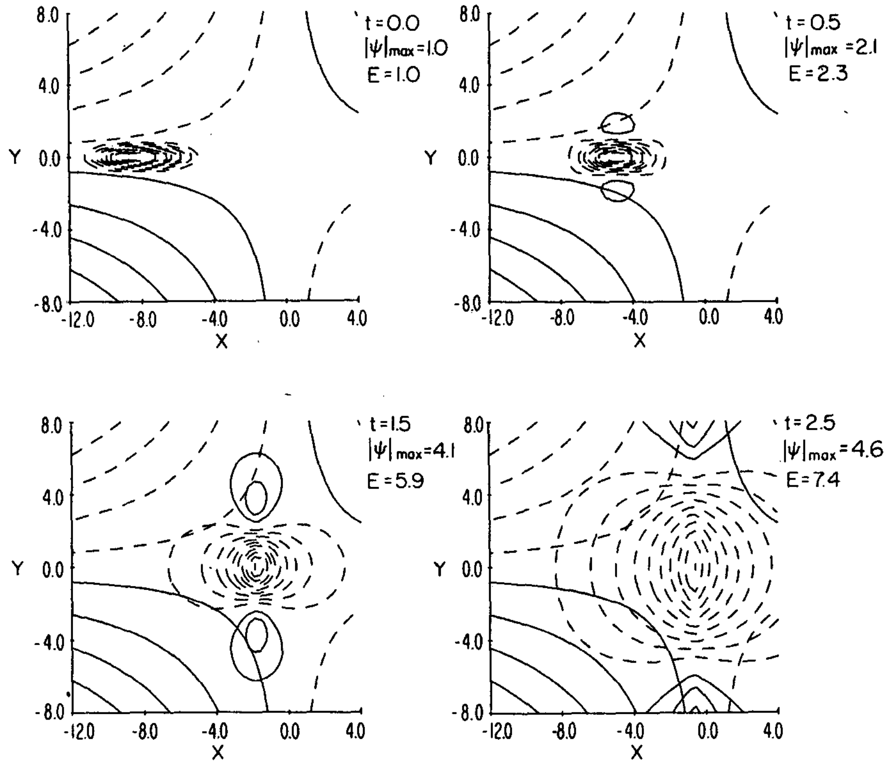

FIG. 2. Development of a zonally elongated perturbation streamfunction in diffluent flow. Maximum amplitude of the streamfunction and perturbation energy, both normalized by their respective values at $t = 0.0$ are indicated as a function of time. Negative contours are dashed and the mean deformation field streamfunction indicated. This perturbation exhibits both growth and scale expansion.

wave with $l = 1.0$, $k = 0.1$; and the zonally elongated initial condition shown in Fig. 2. Exponential nonmodal growth, transient nonmodal growth, and transient growth intercepted by scale expansion are seen in Fig. 3. Rate of energy growth over 2.5 nondimensional time units for the disturbance in diffluence (Fig. 2) is $\Delta \ln E/\Delta t = 0.80$, which is 40% of the maximum allowed by the integral bound (2.6). This rate of development is similar to the maximum rates found in barotropic and baroclinic shear flows (Farrell 1985, 1987).

The synoptic situation that results in rapid development in diffluent flow is a zonally or more generally a streamwise elongation of the initial perturbation, a configuration that can be seen from a study of 500 hPa flows to arise from a number of separately identifiable processes including elongation of a single disturbance in an upstream confluent region (often in northwest flow leading into a diffluent trough), and the linking up of two separate centers in a split flow to form a zonal train with a composite increase in zonal wavelength. It is frequently the case in practice that the occurrence of favorable perturbations is more apparent in the vorticity field than in the height field.

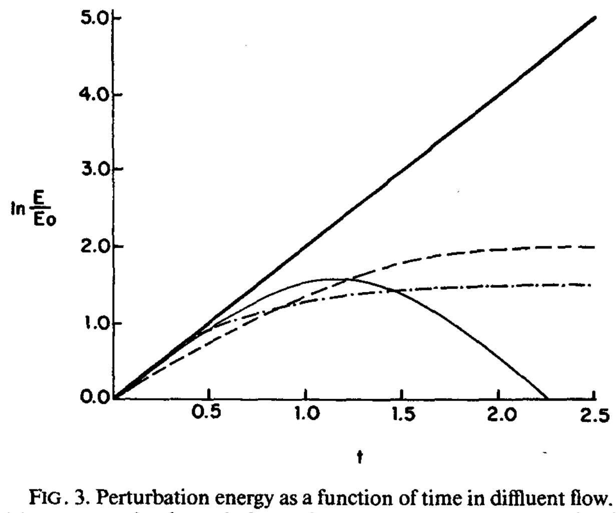

FIG. 3. Perturbation energy as a function of time in diffluent flow. Plane wave with $l = 1.0$, $k = 0.0$, heavy solid. Plane wave with $l = 1.0$, $k = 0.1$, light solid. Zonally elongated Gaussian initial disturbance as in Fig. 2, dash. The perturbation confined to a rectangle in wavenumber with $\delta = 1/2$, dash dot. The plane wave with $k = 0$ grows at the maximum possible rate, the zonally elongated asymmetric Gaussian and the rectangular excitation in wavenumber exhibit both rapid growth and subsequent asymptotic approach to a constant amplitude.

A confluent flow ($\alpha < 0$) requires kinematically,

$$k(t) = k_0 e^{-t}$$
$$l(t) = l_0 e^{t}.$$

Development in confluent flow is illustrated by the perturbation,

$$\psi(x, y; t = 0) = -e^{-[(2(x-1))^2+(y/2)^2]},$$

the evolution of which is shown in Fig. 4.

Regions of confluence are found characteristically, but by no means exclusively, at the upstream end of the climatological storm tracks. The formation of a favorable perturbation often occurs in association with a split flow in the branches of which independent troughs are propagating toward the confluent region formed by the joining of the two (usually northern and southern) branches. The simultaneous arrival of the independently propagating troughs at the confluence produces a composite perturbation that is elongated in the meridional direction so that it undergoes explosive development on advection into the confluence. The challenge to a forecast model posed by this mechanism of growth is severe. Not only must the model accurately advect the troughs over the time of the forecast so that they arrive at the confluence simultaneously so as to make a single composite structure, but the model must also maintain the short zonal wavelength of the troughs so that the zonal to meridional aspect ratio that determines the growth is accurately represented. This is a mechanism of development that is commonly referred to in the synoptic literature as trough phasing, although the reader is cautioned that due at least in part to the lack in the past of a unifying theoretical framework for transient development, this nomenclature is not consistent and the term is sometimes used to refer to the overtaking of a lower level potential vorticity perturbation by an upper tropospheric short wave in a baroclinic shear flow; the classical Petterssen type B cyclogenesis described by Petterssen and Smebye (1971).

The growth of the perturbations shown in Figs. 2 and 4 can be better understood by examining energy as a function of time for an excitation the Fourier spectrum of which is confined to have a constant amplitude over a rectangle in the wavenumber domain. Consider a constant amplitude A_0 over the interval $-\mu_0\delta < k_0 < \mu_0\delta$ and $-(\mu_0/\delta) < l_0 < \mu_0/\delta$, where δ^{-2} is a dimensionless aspect ratio, $x{:}y$. The perturbation is initially of the form:

$$\psi(x, y; t = 0) = 4A_0 \frac{\sin(\mu_0\delta x)}{x} \frac{\sin(\mu_0\delta^{-1} y)}{y}.$$

This perturbation is less localized than the Gaussian but behaves similarly, exhibiting for $\alpha > 0$ and $\delta < 1$

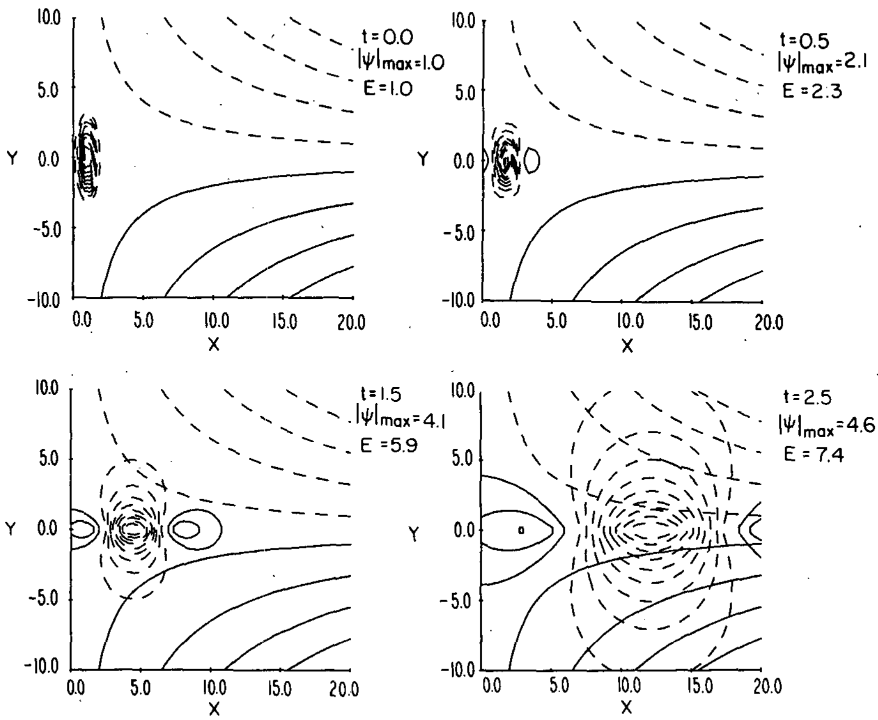

FIG. 4. Development of a meridionally elongated perturbation streamfunction in confluent flow. Maximum amplitude of the streamfunction and perturbation energy, both normalized by their respective values at $t = 0.0$ are indicated. Negative contours are dashed and the mean deformation field streamfunction indicated. This perturbation exhibits both growth and scale expansion.

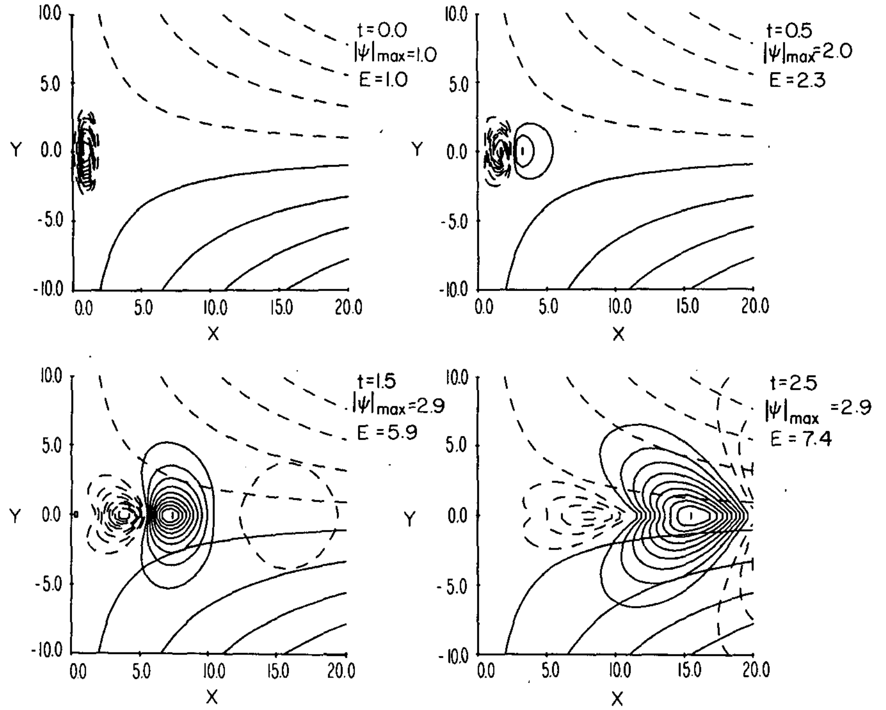

FIG. 5. The same example as in Fig. 4, development in confluent flow, except with the inclusion of the planetary vorticity gradient $\beta = 0.8$. Maximum amplitude of the streamfunction and perturbation energy, both normalized by their respective values at $t = 0.0$ are indicated. A ridge forced by advection of low vorticity fluid ahead of the disturbance center is apparent.

(a zonally elongated disturbance in diffluent flow as in Fig. 2) a rapid initial growth followed by an asymptotic scale expansion at nearly constant amplitude as $t \to \infty$. The energy is

$$E(t) = \frac{A_0^2}{4} \int_{-\mu_0/\delta}^{\mu_0/\delta} \int_{-\mu_0\delta}^{\mu_0\delta} \frac{(k_0^2 + l_0^2)^2}{k_0^2 e^{2t} + l_0^2 e^{-2t}} dk_0 dl_0.$$

This integral can be reduced to

$$E(t) = \frac{A_0^2}{4} \left[\frac{4\mu_0^4 e^{-2t}}{3\delta^2} (\delta^4 + 2 - e^{-4t}) + (2 + 2e^{-8t} - 4e^{-4t}) \int_{-\mu_0/\delta}^{\mu_0/\delta} l_0^3 \tan^{-1}\left(\frac{e^{2t}\mu_0\delta}{l_0}\right) dl_0 \right].$$

As $t \to \infty$ the remaining integral dominates obtaining the asymptotic value

$$\frac{A_0^2 \mu_0^4 \pi}{8\delta^4}.$$

The initial energy is

$$\frac{A_0^2 \mu_0^4}{3} \left(\frac{1}{\delta^2} + \delta^2 \right).$$

The increase in energy is monotone approaching:

$$\frac{E(\infty)}{E(0)} = \frac{3\pi}{8\delta^2} \left(\frac{1}{1 + \delta^4} \right),$$

accompanied by a monotone scale expansion as detailed above, the growth being nearly proportional to aspect ratio for moderately large δ^{-1}. Energy as a function of time for the example above with $\delta = 1/2$ is shown in Fig. 3. Again, localized perturbations result in accumulation of energy in large scales not in algebraic decay as might be expected.

When the gradient of planetary vorticity is included by setting $\beta = 0.8$ advection by the wave of low vorticity fluid from the south rapidly builds a ridge downstream of the center of the disturbance (Fig. 5). This process is commonly observed to occur in regions downstream of oceanic storm tracks and it often results in the development of a blocking ridge.

The rapid and episodic character of transient development argues for tracing the nonlinear extension of the above linear results to direct forcing by the developing disturbance perhaps followed by equilibration as in the examples of Hou and Farrell (1986) for transient development in shear flow. It is suggested that some blocks, rather than being maintained by quadratic flux divergence (Shutts 1983), are instead made up of the waves themselves collapsed in vorticity to frontal scale by the deformation but of the considerably greater block scale in the induced streamfunction, as described by Berggren et al. (1949).

In regard to wave–mean flow interaction a distinction must be made between the perhaps relatively rare synoptically significant major developments resulting from favorable perturbations and the statistical average of all perturbations. The tendency of these statistical averages appears to be to accelerate the mean jet in storm track regions (Lau and Holopainen 1984), which implies that on average perturbations decay rather than grow in the net through barotropic processes.

4. Modeling the effect of divergence

The barotropic vorticity equation applies without approximation to strictly barotropic flow confined by rigid horizontal boundaries and it is customary to regard it as applying at some midtropospheric level of nondivergence when (2.2) is used as a model for atmospheric dynamics. The barotropic vorticity equation is a natural choice for study of fundamental ideas relating to development in confluent and diffluent flow because of the simplicity of its analysis and straightforward physical interpretability of its dynamics. However, the expansion effect that is a consequence of the development process raises questions regarding the lack of an intrinsic scale of influence in this model. In order to include the deformation scale in a simple way the inviscid divergent barotropic vorticity equation is used (Pedlosky 1987):

$$\frac{\partial(\nabla^2\psi - \gamma^2\psi)}{\partial t} + \left[-(\mathrm{sgn}\alpha)x\frac{\partial}{\partial x} + (\mathrm{sgn}\alpha)y\frac{\partial}{\partial y}\right] \times (\nabla^2\psi - \gamma^2\psi) + J[\psi, \Pi_0] = 0. \quad (4.1)$$

The divergence parameter γ^2 is the inverse of the square Rossby radius and the background potential vorticity $\Pi_0 = -\gamma^2 xy + \beta y$ includes a contribution from the deviation of the deformable free surface arising from the requirement of geostrophic balance. This nonseparable spatially varying equivalent beta effect greatly complicates analysis. In order to preserve the simplicity of analytic solutions the freedom to arbitrarily choose a forcing to maintain the base state in a linear problem is invoked and the background deformation flow balanced while leaving the free surface undisturbed in the mean. (Alternatively a bottom topography could be chosen to cancel the surface induced contribution to the equivalent beta effect.) With this simplifying assumption the complex amplitude factor (2.4) becomes

$$F(t) = F_0 \frac{k_0^2 + l_0^2 + \gamma^2}{k(t)^2 + l(t)^2 + \gamma^2} e^{i\beta \int_0^t k(\tau)/k(\tau)^2 + l(\tau)^2 + \gamma^2 d\tau}, \quad (4.2)$$

while the total energy varies similar to (2.5):

$$E(t) = E_0 \frac{k_0^2 + l_0^2 + \gamma^2}{k(t)^2 + l(t)^2 + \gamma^2}. \quad (4.3)$$

A Rossby radius of 1000 km corresponds to $\gamma^2 = 0.25$ and with $\beta = \nu = 0$ the example of Fig. 2, a zonally elongated disturbance entering a region of diffluence that has initially the form (3.1) evolves as in Fig. 6. Divergence is often anticipated to have little effect on perturbations that are small in scale compared to the deformation radius, but the expansion inherent in the dynamics of localized disturbances in deformation results ultimately in all such perturbations being affected by a finite deformation radius. While the rate of growth is curtailed compared to that found in the barotropic example it should be noticed that over the first 1.5 time units, corresponding dimensionally to 42 h, growth proceeds at 50% of the theoretical maximum given by (2.6). This example indicates that the contribution of transient development in deformation flow is likely to be of greater importance on space scales smaller than or comparable to the deformation radius.

5. Discussion and conclusion

To the extent that the mean flow in a channel deviates from uniform translation or the flow on a sphere deviates from solid body rotation there is a nonvanishing element in the rate of strain, and energy integral relations allow for the development of properly configured perturbations. One and two dimensional shear flows are well-known examples associated with the familiar baroclinic, barotropic, and mixed baroclinic–barotropic instability mechanism for the growth of exponential normal modes. The development of perturbations in a flow with nonvanishing rate of strain is, however, a much more general phenomenon than would be the case if growth were limited by the existence and growth rate of exponential normal modes. For shear flows that support instabilities properly configured perturbations grow at rates that exceed that of the exponential modes, often by large factors (Farrell 1988, 1989). In shear flows stabilized by a short-wave cut-off or by damping, favorable perturbations can be found that exhibit transient growth at rates similar to that in the unstabilized flow (Farrell 1985). These examples have demonstrated the generality with which perturbations can exploit the available energy in shear flow, and in this work attention has turned to the available energy in the component of strain associated with confluence and diffluence. The example used, that of a pure deformation, has the advantages of modeling a local flow of frequent occurrence in the atmosphere and of having analytic solutions. A subset of perturbations in this flow was shown to develop rapidly, even exponentially, despite the fact that the flow does not support normal modes.

Time scales for growth in realistic local deformation in the atmosphere were found to be comparable to observed development times and the form of the perturbation favorable for growth in both confluent and diffluent flow identified. These are high aspect ratio disturbances, zonally elongated in the case of diffluent flow and meridionally elongated in confluent flow.

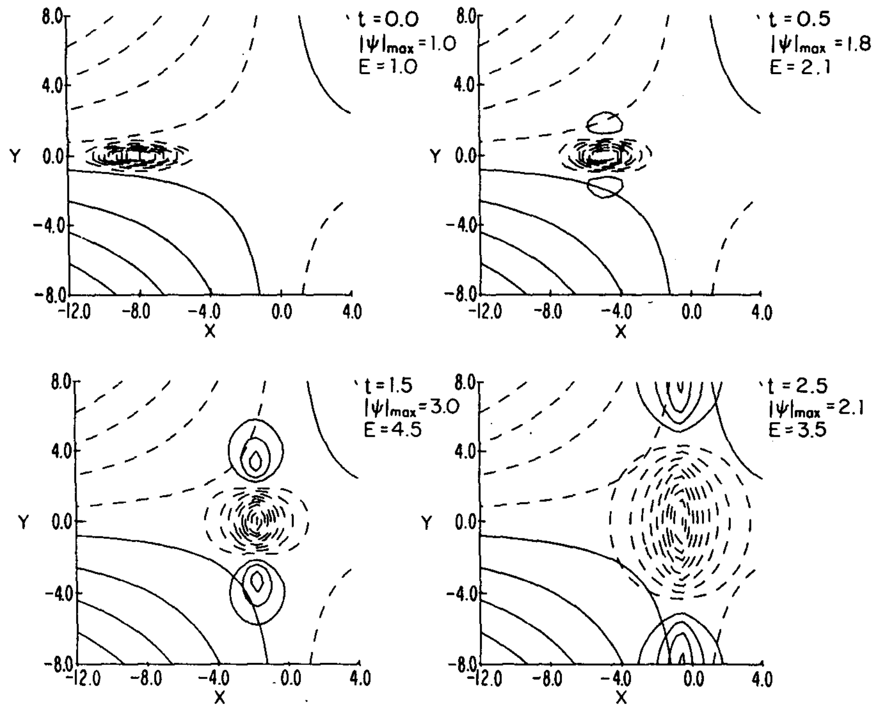

FIG. 6. Development of a zonally elongated perturbation streamfunction in diffluent flow as in Fig. 2 but with the inclusion of a divergence induced deformation scale corresponding to $\gamma^2 = 0.25$. Maximum amplitude of the streamfunction and perturbation energy, both normalized by their respective values at $t = 0.0$ are indicated as a function of time. Negative contours are dashed and the mean deformation field streamfunction indicated.

In addition it was found that a necessary consequence of transient development of spatially localized perturbations in both confluence and diffluence is an expansion in scale of the perturbation that limits decay of disturbances even in the linear limit where nonlinear equilibration and occlusion do not occur.

While much remains to be done, particularly with regard to nonlinear aspects, it is clear that broadening conceptual bounds to embrace the multitude of scenarios associated with transient growth allows a more comprehensive theoretical understanding of development processes in the atmosphere.

Acknowledgments. This work was supported by NSF Grant ATM-8712995.

REFERENCES

Berggren, R., B. Bolin and C.-G. Rossby, 1949: An aerological study of zonal motion, its perturbation and breakdown. *Tellus,* **2,** 14–37.

Boyd, J. P., and Z. D. Christidis, 1987: The algebraic decay of equatorial Rossby waves in a shear flow. *Dyn. Atmos. Oceans,* **11,** 139–151.

Branstator, G., 1985: Analysis of general circulation model sea-surface temperature anomaly simulations using a linear model. Part II: eigenanalysis. *J. Atmos. Sci.,* **42,** 2242–2254.

Bretherton, F. P., and D. B. Haidvogel, 1976: Two-dimensional turbulence above topography. *J. Fluid. Mech.,* **78,** 129–154.

Charney, J. G., 1947: The dynamics of long waves in a baroclinic westerly current. *J. Meteor.,* **4,** 135–162.

Craik, D. D., and W. O. Criminale, 1986: Evolution of wavelike disturbances in shear flows: a class of exact solutions of the Navier–Stokes equations. *Proc. Roy. Soc. London,* **A406,** 13–26.

Eady, E. T., 1949: Long waves and cyclone waves. *Tellus,* **1,** 33–52.

Farrell, B., 1982: The initial growth of disturbances in a baroclinic flow. *J. Atmos. Sci.,* **39,** 1663–1686.

——, 1984: Modal and nonmodal baroclinic waves. *J. Atmos. Sci.,* **41,** 668–673.

——, 1985: Transient growth of damped baroclinic waves. *J. Atmos. Sci.,* **42,** 2718–2727.

——, 1987: Developing disturbances in shear. *J. Atmos. Sci.,* **44,** 2191–2199.

——, 1988: Optimal excitation of neutral Rossby waves. *J. Atmos. Sci.,* **45,** 163–172.

——, 1989: Optimal excitation of baroclinic waves. *J. Atmos. Sci.,* **46,** 1193–1206.

Frederiksen, J. S., 1979: The effects of long planetary waves on the regions of cyclogenesis: linear theory. *J. Atmos. Sci.,* **36,** 737–745.

Fyfe, J., and J. Derome, 1987: The stability of a two-dimensional steady-state flow: local growth of perturbations. *Geophys. Astrophys. Fluid Dyn.,* **39,** 261–294.

Holopainen, E. O., and L. Rontu, 1981: On shear lines in the upper troposphere over Europe. *Tellus,* **33,** 351–359.

Hoskins, B. J., I. N. James and G. H. White, 1983: The shape, propagation and mean-flow interaction of large-scale weather systems. *J. Atmos. Sci.,* **40,** 1595–1612.

Hou, A. Y., and B. Farrell, 1986: Excitation of nearly steady finite-amplitude barotropic waves. *J. Atmos. Sci.,* **43,** 720–728.

Kuo, H. L., 1949: Dynamic instability of two-dimensional non-divergent flow in a barotropic atmosphere. *J. Meteor.*, **6,** 105–122.

Lau, N., and E. O. Holopainen, 1984: Transient eddy forcing of the time-mean flow as identified by geostrophic tendencies. *J. Atmos. Sci.*, **41,** 313–328.

Merkine, L., and R. Balgovind, 1983: Barotropic instability of weakly non-parallel zonal flows. *Geophys. Astrophys. Fluid Dyn.*, **25,** 157–290.

Niehaus, M. C. W., 1980: Instability of non-zonal baroclinic flows. *J. Atmos. Sci.*, **37,** 1447–1463.

Orr, W. M'F., 1907: The stability or instability of the steady motions of a perfect liquid and of a viscous liquid. Parts I and II. *Proc. Roy. Irish Acad.*, **A27,** 9–138.

Pedlosky, J., 1987: *Geophysical Fluid Dynamics,* Springer–Verlag, 710 pp.

Petterssen, S., and S. Smebye, 1971: On the development of extra-tropical cyclones. *Quart. J. Roy. Meteor. Soc.*, **97,** 457–482.

Pierrehumbert, R. T., 1983: Bounds on the growth of perturbations to non-parallel flow on the barotropic beta plane. *J. Atmos. Sci.*, **40,** 1207–1217.

——, 1984: Local and global baroclinic instability of zonally varying flow. *J. Atmos. Sci.*, **41,** 2141–2162.

Shapiro, M. A., 1976: The role of turbulent heat flux in the generation of potential vorticity in the vicinity of upper-level jet stream systems. *Mon. Wea. Rev.*, **104,** 892–906.

Shepherd, T. G., 1985: Time development of small disturbances to plane Couette flow. *J. Atmos. Sci.*, **42,** 1868–1871.

Shutts, G. J., 1983: The propagation of eddies in diffluent jetstreams: eddy vorticity forcing of 'blocking' flow fields. *Quart. J. Roy. Meteor. Soc.*, **109,** 737–761.

Simmons, A. J., J. M. Wallace and G. W. Branstator, 1983: Barotropic wave propagation and instability, and atmospheric teleconnection patterns. *J. Atmos. Sci.*, **40,** 1363–1392.

Thomson, W., 1887: Stability of fluid motion-rectilinear motion of viscous fluid between two parallel planes. *Phil. Mag.*, **24,** 188–196.